10-2B Arithmetic Series

- *find sums of arithmetic sequeneces*
- *A.CED.4*

series: the sum of the terms of a sequence.

arithmetic series: the sum of the terms of an arithmetic sequence.

sum of an arithmetic series: the sum, S_n, of the first ***n*** terms of an arithmetic series is given by $S_n = \frac{n}{2}(a_1 + a_n)$.

$a_n = a_1 + (n-1)d$

Examples: ***Find the sum of the following.***

1. 1st 100 positive integers.

$a_1 = 1 \quad n = 100 \quad a_n = 100$

$S_{100} = \frac{100}{2}(1+100)$

$S_{100} = 50(101)$

$S_{100} = 5050$

2. 1st 10 odd integers.

$a_1 = 1$

$a_{10} = 19$

$n = 10$

1, 3, 5, 7, 9, 11, 13, 15, 17, 19

20 20 20 20

$S_{10} = \frac{10}{2}(1+19)$

$S_{10} = 5(20)$

$S_{10} = 100$

22. $a_9 = 45$

$d = -3$

$a_n = a_1 + (n-1) \cdot d$

20. 24, 35, 46

$a_1 = 24$

$d = 11$

$a_n = 24 + (n-1) \cdot 11$

$a_n = 24 + 11n - 11$

$a_n = 13 + 11n$

38. 182, —, —, —, —, —, 104

$a_1 = 182$

$a_n = 104$

$n = ?$

$104 = 182 + (7-1) \cdot d$

$104 = 182 + 6d$

Examples: ***Find the sum of the following.***

3. The first 50 terms of the arithmetic series where $a_1 = 5$ and $d = 25$.

$a_n = a_1 + (n-1) \cdot d$

$a_{50} = 5 + (50-1)25$

$a_{50} = 5 + 49 \cdot 25$

$a_{50} = 5 + 1225$

$a_{50} = 1230$

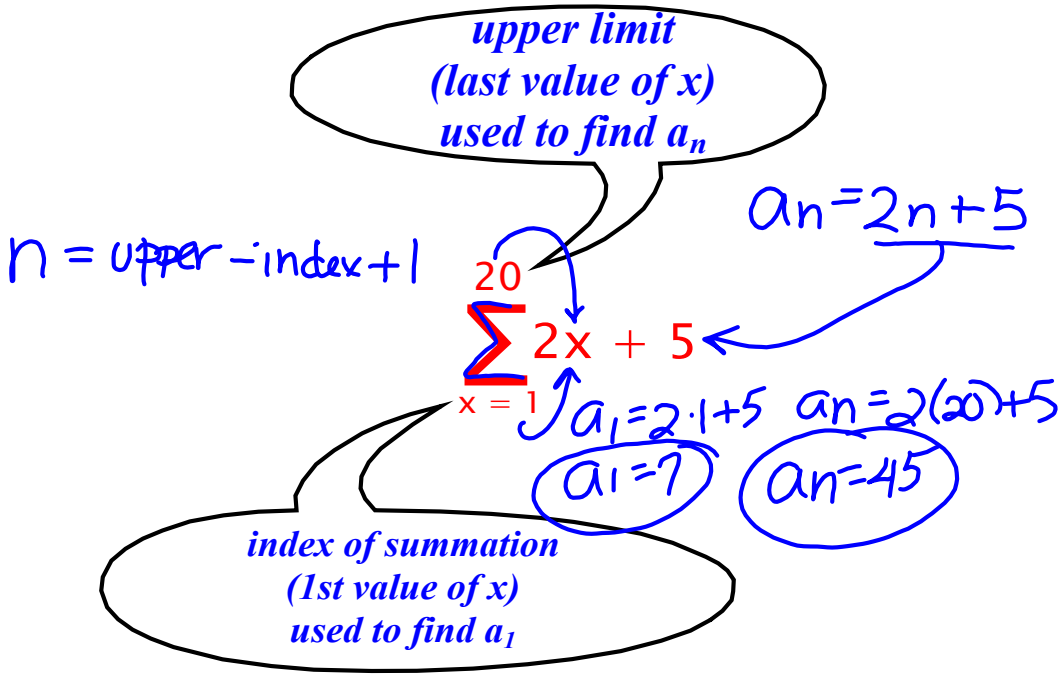
$S_n = \frac{n}{2}(a_1 + a_n)$

$S_{50} = \frac{50}{2}(5 + 1230)$

$S_{50} = 25(1235)$

$S_{50} = 30,875$

sigma or summation notation: abbreviated form of writing a series. Successively replace the index of summation until the upper limit.



Examples: ***Write each series in expanded form and find the sum.***

4. $\sum_{n=1}^{6} 3n+7$

5. $\sum_{k=11}^{24} 3k-6$

a_1 a_2 a_3 a_4 a_5 a_6

$(3 \cdot 1 + 7) + (3 \cdot 2 + 7) + (3 \cdot 3 + 7) + (3 \cdot 4 + 7) + (3 \cdot 5 + 7) + (3 \cdot 6 + 7)$

$10 + 13 + 16 + 19 + 22 + 25$

35

35

105

5. $\sum_{k=11}^{24} 3k-6$

$n = 24-11+1$

$n = 14$

$S_{14} = \frac{14}{2}(27+66)$

$S_{14} = 7(93)$

$S_{14} = 651$

$a_1 = 3 \cdot 11 - 6$

$a_1 = 27$

$a_n = 3 \cdot 24 - 6$

$a_n = 66$

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40-60 evens

$1 + 4 + 7 + \ldots + 43$