10-3B Geometric Series

sum of a geometric series: the sum, S_n, of the first n terms of a geometric series is given by the following formulas:

$S_n = \frac{n}{2}(a_1 + a_n)$

$a_n = a_1 + (n-1)d$

(when a_n is unkn)

$$S_n = \frac{a_1 - a_1 r^n}{1 - r}$$

$a_n = a_1 \cdot r^{n-1}$

$50 = 2(3)^{n-1}$

$25 = 3^{n-1}$

$\log 25 = \log 3^{n-1}$

or

(when n is unknown)

$$S_n = \boxed{\frac{a_1 - a_n r}{1 - r}}$$

how do we get this new formula?

$x^3 \cdot x^4 = x^?$

$$r^1 \cdot a_n = a_1 r^{n-1} \cdot r^1$$

$$a_n \cdot r = a_1 r^n$$

$S_n = \frac{a_1 - a_1 r^n}{1 - r}$ becomes $S_n = \frac{a_1 - a_n \cdot r}{1 - r}$

Examples

Find the sum of the following geometric series.

$16 + 8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$

1. $a_1 = 16$, $r = \frac{1}{2}$, $n = 7$

$$S_7 = \frac{a_1 - a_1 \cdot r^n}{1 - r}$$

$$S_7 = \frac{16 - 16\left(\frac{1}{2}\right)^7}{1 - \frac{1}{2}}$$

$$S_7 = \frac{16 - 16\left(\frac{1}{128}\right)}{\frac{1}{2}}$$

$$S_7 = \frac{\left(16 - \frac{1}{8}\right) \cdot 2}{\cancel{\frac{1}{2}} \cdot 2}$$

$$S_7 = 32 - \frac{1}{4}$$

$$S_7 = 31\tfrac{3}{4}$$

$$S_7 = 31.75$$

2. $a_1 = 4$, $a_n = 256$, $r = 4$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{4 - 256(4)}{1 - 4}$$

$$S_n = \frac{4 - 1024}{-3}$$

$$S_n = \frac{-1020}{-3} = 340$$

3. Find a_1 for the series if $S_8 = 13{,}120$ and $r = 3$.

n

$a_1 = ?$

$r = 3$

$n = 8$

$$S_n = \frac{a_1 - a_1 \cdot r^n}{1 - r}$$

$$13{,}120 = \frac{a_1 - a_1 \cdot 3^8}{1 - 3}$$

$$13{,}120 = \frac{1a_1 - 6561a_1}{-2}$$

$$-2 \cdot 13{,}120 = \frac{-6560a_1}{\cancel{-2}} \cdot -2$$

$$-26{,}240 = -6560a_1$$

$$4 = a_1$$

$x - 7x$

$-6x$

Find the sum of the following series.

4. $$\sum_{n=1}^{12} 3(2)^{n-1}$$

$a_1 = 3 \cdot 2^0 = 3$

$n = 12$

$r = 2$

$S_{12} = \frac{a_1 - a_1 \cdot r^n}{1-r}$

$S_{12} = \frac{3 - 3(2)^{12}}{1-2}$

$S_{12} = \frac{3 - 3(4096)}{-1}$

$S_{12} = \frac{3 - 12{,}288}{-1}$

$S_{12} = \frac{-12{,}285}{-1}$

$\boxed{S_{12} = 12{,}285}$

5. $$\sum_{t=5}^{8} 2(4)^{t-1}$$

$a_1 = 2(4)^{5-1} = 512$

$r = 4$

$n = 8-5+1$

$n = 4$