

12-3A Trigonometric Functions

evaluate trig. functions for a given point

previous assignment:
Degrees & Radians WS

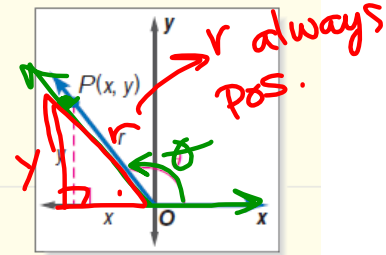
θ'

Key Concept Trigonometric Functions, θ in Standard Position

Let θ be an angle in standard position and let $P(x, y)$ be a point on the terminal side of θ . Using the Pythagorean Theorem, the distance r from the origin to P is given by $r = \sqrt{x^2 + y^2}$. The trigonometric functions of an angle in standard position may be defined as follows.

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}, x \neq 0$$

$$\csc \theta = \frac{r}{y}, y \neq 0 \quad \sec \theta = \frac{r}{x}, x \neq 0 \quad \cot \theta = \frac{x}{y}, y \neq 0$$



To evaluate trigonometric functions given a point P:

- draw the angle in standard position and label point P on the terminal side of the angle.
- find r (*hypotenuse*) using the pythagorean theorem.
- use the x , y , and r values to evaluate the trig. functions.
- we are finding the trig. functions for θ , but we are using the *acute angle* with the x -axis.

Examples: Find the exact values of the 6 trigonometric functions of θ if the terminal side of θ contains the given point P.

1. $P(8, -15)$

$$8^2 + (-15)^2 = r^2$$

$$64 + 225 = r^2$$

$$\sqrt{289} = \sqrt{r^2}$$

$$17 = r$$

$$\sin \theta = \frac{-15}{17} \quad \csc \theta = \frac{17}{-15}$$

$$\cos \theta = \frac{8}{17} \quad \sec \theta = \frac{17}{8}$$

$$\tan \theta = \frac{-15}{8} \quad \cot \theta = \frac{-8}{15}$$

2. $P(4, 4)$

$$(-10)^2 + 8^2 = r^2$$

$$100 + 64 = r^2$$

$$\sqrt{164} = \sqrt{r^2}$$

$$2\sqrt{41} = r$$

$$\sin \theta = \frac{8}{2\sqrt{41}} = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$$

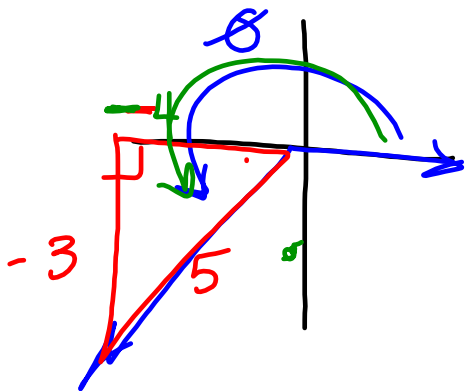
$$\cos \theta = \frac{-10}{2\sqrt{41}} = \frac{-5}{\sqrt{41}} = \frac{-5\sqrt{41}}{41}$$

$$\tan \theta = \frac{8}{-10} = \frac{-4}{5}$$

$$\csc \theta = \frac{\sqrt{41}}{4}$$

$$\sec \theta = \frac{-\sqrt{41}}{5} \quad \cot \theta = \frac{-5}{4}$$

6. Suppose θ is an angle in standard position whose terminal side is in quadrant III and $\csc \theta = \frac{-5}{3}$. Find the exact values of the remaining five trigonometric functions.



$$\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{-5}{3} = \frac{5}{-3}$$

$$\sin \theta = \frac{-3}{5}$$

$$\cos \theta = \frac{-4}{5}$$

$$\tan \theta = \frac{3}{4}$$

$$\sec \theta = \frac{-5}{4}$$

$$\cot \theta = \frac{4}{3}$$

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(skip quadrants) $(0, y)$

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skip $(x, 0)$

Attachments

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