

13-1A Trigonometric Identities

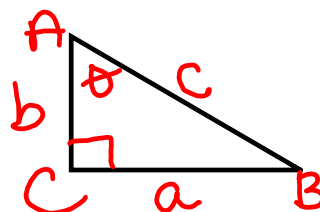
Trigonometric Identity: an equation involving trigonometric functions that is true for all values for which every expression in the equation is defined.

| Trigonometric Identities | | |
|--|---|-------------------------------------|
| ÷ Quotient Identities | $\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$ | |
| Reciprocal Identities | $\csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$ | |
| $\cos^2 \theta = (\cos \theta)^2 \neq \cos \theta^2$ Pythagorean Identities | | |
| $\cos^2 \theta + \sin^2 \theta = 1$ | $\tan^2 \theta + 1 = \sec^2 \theta$ | $\cot^2 \theta + 1 = \csc^2 \theta$ |

Trigonometric Identities - prove it!
Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

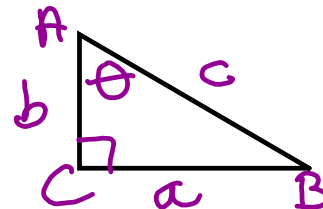


$$\frac{b}{a} = \frac{\frac{b}{c}}{\frac{a}{c}}$$

$$\frac{b}{a} = \frac{b}{a}$$

$$\frac{b}{c} \div \frac{a}{c} = \frac{b}{a}$$

Trigonometric Identities - prove it!
Reciprocal Identities



$$\text{csc } \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

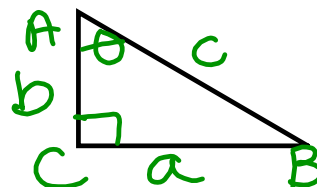
$$\frac{c}{a} = \frac{1}{\frac{a}{c}}$$

$$1 \div \frac{a}{c}$$

$$1 \cdot \frac{c}{a}$$

$$\frac{c}{a} = \frac{c}{a}$$

Trigonometric Identities - prove it!
Pythagorean Identities



$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\left(\frac{a}{b}\right)^2 + 1 = \left(\frac{c}{b}\right)^2$$
~~$$\frac{a^2}{b^2} + 1 = \frac{c^2}{b^2}$$~~

$$a^2 + b^2 = c^2$$

Using trig. identities to find a value

1. Find $\tan\theta$ if $\sec\theta = -2$ and $180^\circ < \theta < 270^\circ$

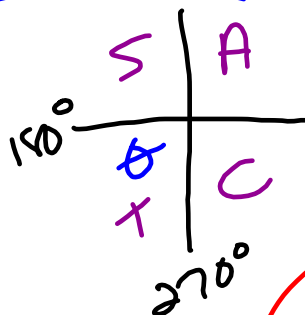
$$\tan^2\theta + 1 = \sec^2\theta$$

$$\tan^2\theta + 1 = (-2)^2$$

$$\tan^2\theta + 1 = 4$$

$$\tan^2\theta = 3$$

$$\tan\theta = \pm\sqrt{3}$$



$$x^2 + 1 = 4$$

2. Find $\sin\theta$ if $\cos\theta = 1/2$ and $270^\circ < \theta < 360^\circ$

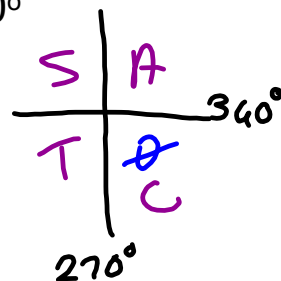
$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{1}{2}\right)^2 + \sin^2\theta = 1$$

$$\frac{1}{4} + \sin^2\theta = \frac{4}{4}$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{3}{4}}$$

$$\sin\theta = -\frac{\sqrt{3}}{2}$$



3. Find $\csc\theta$ if $\cos\theta = \frac{2}{3}$ and $90^\circ < \theta < 180^\circ$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(\frac{2}{3}\right)^2 + \sin^2\theta = 1$$

$$\frac{4}{9} + \sin^2\theta = 1 \quad | -\frac{4}{9}$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{5}{9}}$$

$$\sin\theta = \frac{\sqrt{5}}{3}$$

$\csc\theta = \frac{1}{\sin\theta}$
 $\csc\theta = \frac{1}{\frac{\sqrt{5}}{3}}$
 $\csc\theta = \frac{3}{\sqrt{5}}$
 $\csc\theta = \frac{3\sqrt{5}}{5}$

How to use trigonometric identities to find exact trig. values

- Determine which trig. identity/identities to use.
 - Is there an identity with function & function? 1 identity
 - Is there an identity with function & reciprocal? 2 id's
 - Is there an identity with reciprocal & function? 2 id's
 - Is there an identity with reciprocal & reciprocal? 3 id's
- Use the trig. identity/identities as formulas.
- Use substitution and solve.
- Determine the sign of the trig. value based on the quadrant.

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10. $\text{csc } \theta = \frac{1}{\sin \theta}$

$$\cot^2 \theta + 1 = \csc^2 \theta$$

$$\cot \theta = \frac{1}{\tan \theta}$$