

13-1B Trigonometric Identities

Using trig. identities to simplify an expression:

- Look for a direct relationship between trigonometric functions and relationships between inverse trigonometric functions.
- Simplify by replacing trigonometric functions with identities.
- Expressions will usually simplify to a numeric value OR a single trigonometric function.

Simplify.

1. $\sin\theta(\csc\theta - \sin\theta)$

$$\sin\theta \cdot \csc\theta - \sin^2\theta$$

$$\frac{\cancel{\sin\theta} \cdot 1}{\cancel{\sin\theta}} - \sin^2\theta$$

$$\boxed{1 - \sin^2\theta}$$
$$\boxed{\cos^2\theta}$$

$$\theta = 45^\circ$$

$$\left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$$

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = 1$$

$$\csc 45^\circ = \sqrt{2}$$

$$\sec 45^\circ = \sqrt{2}$$

$$\cot 45^\circ = 1$$

$$\sin 45^\circ (\csc 45^\circ - \sin 45^\circ)$$

$$\frac{\sqrt{2}}{2} \left(\frac{\sqrt{2}}{1} - \frac{\sqrt{2}}{2} \right)$$

$$\frac{\sqrt{2}}{2} - \frac{2}{4} = \frac{1}{2}$$

2. $\csc\theta\cos\theta\tan\theta$

$$\left(\frac{1}{\cancel{\sin\theta}}\right) \left(\frac{\cancel{\cos\theta}}{1}\right) \left(\frac{\cancel{\sin\theta}}{\cancel{\cos\theta}}\right)$$

$$\textcircled{1}$$

$\theta = 30^\circ$

$$(\csc 30)(\cos 30)(\tan 30)$$

$$\frac{2}{1} \left(\frac{\sqrt{3}}{2}\right) \left(\frac{\sqrt{3}}{3}\right) = \frac{2}{1} \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{3} = \frac{2}{1} \cdot \frac{3}{6} = \frac{2}{1} \cdot \frac{1}{2} = 1 = \textcircled{1}$$

3. $\csc\theta\tan\theta$

$$\frac{1}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{\cos\theta}$$

$$\frac{1}{\cos\theta}$$
$$\boxed{\sec\theta}$$

$$(\tan\theta)^2 = \left(\frac{\sin\theta}{\cos\theta}\right)^2$$

$$\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$(\csc\theta)^2 = \left(\frac{1}{\sin\theta}\right)^2$$

$$\csc^2\theta = \frac{1}{\sin^2\theta}$$

4. $\frac{\csc^2 \theta - \cot^2 \theta}{1 - \cos^2 \theta}$

$$\frac{1}{\sin^2 \theta}$$

$$\boxed{\csc^2 \theta}$$

$$\frac{1}{\frac{1}{5} + \frac{3}{5}}$$

$$\frac{1}{\frac{4}{5}}$$

$$\frac{\csc^2 \theta}{1 - \cos^2 \theta} - \frac{\cot^2 \theta}{1 - \cos^2 \theta}$$

$$\frac{\csc^2 \theta}{\sin^2 \theta} - \frac{\cot^2 \theta}{\sin^2 \theta}$$

$$\frac{\frac{1}{\sin^2 \theta}}{\sin^2 \theta} - \frac{\frac{\cos^2 \theta}{\sin^2 \theta}}{\sin^2 \theta}$$

$$\frac{1}{\sin^4 \theta} = \frac{1}{\sin^2 \theta} \cdot \frac{1}{\sin^2 \theta}$$

$$\frac{\cos^2 \theta}{\sin^4 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} \cdot \frac{1}{\sin^2 \theta}$$

$$\frac{1}{\sin^4 \theta} - \frac{\cos^2 \theta}{\sin^4 \theta}$$

$$\frac{1 - \cos^2 \theta}{\sin^4 \theta}$$

$$\frac{\sin^2 \theta}{\sin^4 \theta} = \frac{1}{\sin^2 \theta}$$

$$\boxed{\csc^2 \theta}$$

13-1B Worksheet