

## 3-4 Solving Systems of Equations

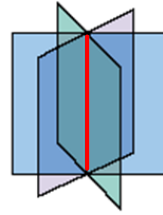
### 3 unknowns

- 3 unknowns requires 3 equations
- 3 variables graph as planes, not lines.
- use a combination of elimination and substitution to solve.
- show DETAILED work, make notes throughout.

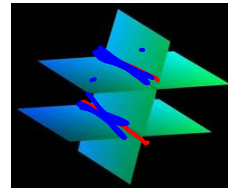
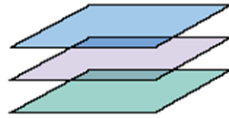
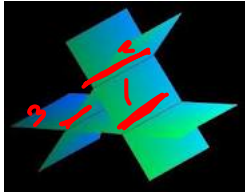
#### One Solution (x, y, z)



#### Infinitely Many Solutions



#### No Solutions



$$12y = (x+3)^2$$
$$y = (-2x-3)$$

$$-2x-3 = (x+3)^2$$

$$-2x-3 = x^2+6x+9$$

$$0 = x^2+8x+12$$

$$0 = (x+6)(x+2)$$

$$x+6=0$$

$$x=-6$$

$$(-6, 9)$$

$$x+2=0$$

$$x=-2$$

$$(-2, 1)$$

Examples: Solve the following systems of equations.  $(-2, 6, -3)$

1.  $5x + 3y + 2z = 2$

$2x + y - z = 5$

$x + 4y + 2z = 16$

$-2 + 4(6) + 2z = 16$

$-2 + 24 + 2z = 16$

$22 + 2z = 16$

$2z = -6$

$z = -3$

$5x + 3y + 2z = 2$

$\text{elim } z \quad -x - 4y - 2z = -16$

$4x - y = -14$

elim z

$5x + 3y + 2z = 2$

$+ 4x + 2y - 2z = 10$

$9x + 5y = 12$

$9(-2) + 5y = 12$

$-18 + 5y = 12$

$5y = 30$   
 $y = 6$

$20x + 5y = -70$

$9x + 5y = 12$

$29x = -58$

$x = -2$

Examples: Solve the following systems of equations.

2.  $3x - y - 2z = 4$   
 $6x - 2y - 4z = 11$   
 $9x - 3y - 6z = 12$

~~-2~~ ~~elim y~~

~~$-6x + 2y + 4z = -8$~~   
 ~~$6x - 2y - 4z = 11$~~   


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 $0 \neq 3$   
 $\emptyset$

$(1 \times 2) \quad 5x + 3z = 4$   
 $(2 \times 3) \quad 5x + 3z = 4$   
 $(1 \times 3) \quad 5x + 3z = 4$  }  $\infty$  many

Examples: Solve the following systems of equations.

$(1, 1, 1)$

3.  $x + 2y + z = 4$

$4y - 3z = 1$   
 $y + 5z = 6$

~~4~~ ~~elim y~~

$4y - 3z = 1$   
 $-4y - 20z = -24$   


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 $-23z = -23$

$y + 5 = 6$   
 $y = 1$

$z = 1$

$x + 2 + 1 = 4$   
 ~~$x + 3 = 4$~~   
 $x = 1$

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## 9 - 17 odd

$$\begin{aligned} 2x + 3y - 4z &= 10 \\ \boxed{x - 3z} &= 5 \\ 3x + y + 2z &= -13 \end{aligned}$$

dim y

Solve each system of equations.

8.  $-5x + y - 4z = 60$   
 $2x + 4y + 3z = -12$   
 $6x - 3y - 2z = -52$   
 **$(-8, 4, -4)$**
11.  $4r + 6s - t = -18$   
 $3r + 2s - 4t = -24$   
 $-5r + 3s + 2t = 15$   
 **$(-2, -1, 4)$**
14.  $8x + 3y + 6z = 43$   
 $-3x + 5y + 2z = 32$   
 $5x - 2y + 5z = 24$   
 **$(-1, 3, 7)$**
17.  $2x - y + z = 1$   
 $x + 2y - 4z = 3$   
 $4x + 3y - 7z = -8$   
**no solution**
9.  $4a + 5b - 6c = 2$   
 $-3a - 2b + 7c = -15$   
 $-a + 4b + 2c = -13$   
 **$(-3, -2, -4)$**
12.  $-2x + 15y + z = 44$   
 $4x + 3y + 3z = 18$   
 $-3x + 6y - z = 8$   
**no solution**
15.  $-6x - 5y + 4z = 53$   
 $5x + 3y + 2z = -11$   
 $8x - 6y + 5z = 4$   
 **$(-4, -1, 6)$**
18.  $x + 2y = 12$   
 $3y - 4z = 25$   
 $x + 6y + z = 20$   
 **$(6, 3, -4)$**
10.  $-2x + 5y + 3z = -25$   **$(8, -3, 2)$**   
 $-4x - 3y - 8z = -39$   
 $6x + 8y - 5z = 14$
13.  $4x + 2y + 6z = 13$  **infinite solutions**  
 $-12x + 3y - 5z = 8$   
 $-4x + 7y + 7z = 34$
16.  $-9a + 3b - 2c = 61$   
 $8a + 7b + 5c = -138$   
 $5a - 5b + 8c = -45$   
 **$(-8, -7, -5)$**
19.  $r - 3s + t = 4$   
 $3r - 6s + 9t = 5$   
 $4r - 9s + 10t = 9$   
**infinite solutions**
- 20b. 7 swimmers placed third, 5 swimmers placed second, and 12 swimmers placed first.**

13.  $4x + 2y + 6z = 13$  infinite solutions

$-12x + 3y - 5z = 8$

$-4x + 7y + 7z = 34$

$4x + 2y + 6z = 13$   
 $-4x + 7y + 7z = 34$

$9y + 13z = 47$

$12x + 6y + 18z = 39$   
 $+ -12x + 3y - 5z = 8$

$9y + 13z = 47$

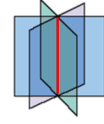
$-12x + 3y - 5z = 8$   
 $+ 12x - 21y - 21z = -102$   


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 $-18y - 26z = -94$   
 $-2 \quad \quad \quad -2$   


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 $9y + 13z = 47$



17.  $2x - y + z = 1$   
 $x + 2y - 4z = 3$   
 $4x + 3y - 7z = -8$   
 no solution

② elim y  $4x - 2y + 2z = 2$   
 $+ x + 2y - 4z = 3$   


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 $5x - 2z = 5$

elim y  $6x - 3y + 3z = 3$   
 $+ 4x + 3y - 7z = -8$   


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 $10x - 4z = -5$

$10x + 4z = -10$   
 $10x - 4z = -5$   


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 $0 = -15$   
 $\emptyset$

No Solutions

