4-1 Graphing Quadratic Functions

- *graph quadratic functions.*
- *interpret the graph and significant parts of the graph*
- *find and interpret the maximum or minimum values of a quadratic function*
- *A.SSE.1a, F.IF.9*

quadratic function: a function described by an equation

$$f(x) = ax^2 + bx + c, \text{ where } a \neq 0.$$

quadratic term

linear term

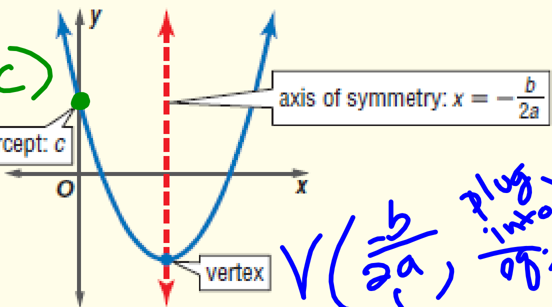
constant

the graph of any quadratic function is called a **parabola**.

Key Concept *Graph of a Quadratic Function*

- **Words** Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.
  - The y-intercept is $\cancel{a(0)^2 + b(0)} + c$ or c.
  - The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
  - The x-coordinate of the vertex is $-\frac{b}{2a}$.
- **Model**

$(0, c)$



$x = \frac{-b}{2a}$

$V\left(\frac{-b}{2a}, \text{plug into og.}\right)$

Graphing Quadratic Functions

- find the vertex, y-intercept, axis of symmetry.
- make a table of *at least* 5 points.
- plot and label all info, sketch the parabola.

x	y
(0	c)
(	)
v (x	y)
(	)
(x	y)

use symm.

Example: ***Find the y-intercept, equation of the axis of symmetry, and the vertex for the function. Then graph the function.***
$f(x) = x^2 + 8x + 9$.

$$f(x) = ax^2 + bx + c$$

y-int: (0, 9)

$x = \frac{-b}{2a}$

$x = \frac{-8}{2(1)}$

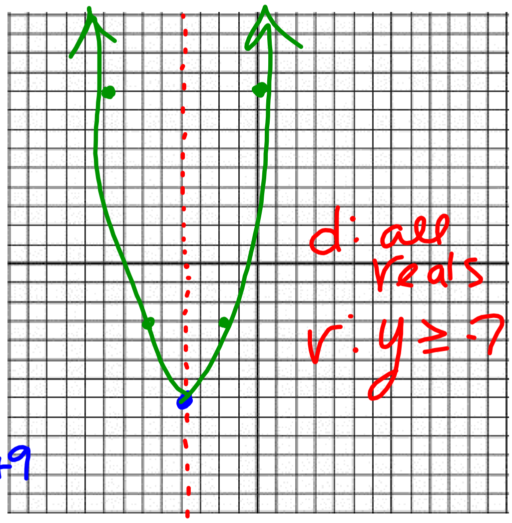
$\boxed{x = -4}$

X	Y
(-8	9)
(-6	-3)
v (-4	-7)
(-2	-3)
(0	9)

$y = (-4)^2 + 8(-4) + 9$

$y = 16 - 32 + 9$

$y = -7$



$y = (-2)^2 + 8(-2) + 9$

$4 - 16 + 9$

$y = -3$

Example: ***Graph and label the function f(x) = x² + 3x - 1***

$y = x^2 + 3x - 1$

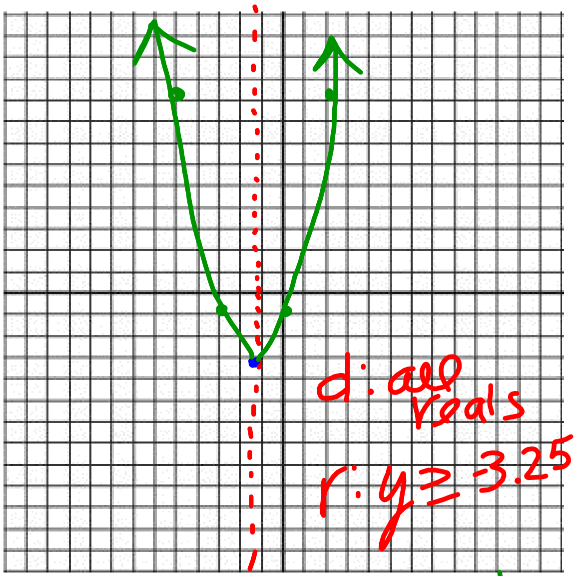
y-int: (0, -1)

$x = \frac{-b}{2a}$

$x = \frac{-3}{2(1)}$

$x = -1.5$ axis of symm.

X	Y
(-5	9)
(-3	-1)
(-1.5	-3.25)
(0	-1)
(2	9)



$y = \left(-\frac{3}{2}\right)^2 + 3\left(\frac{-3}{2}\right) - 1$

$\frac{9}{4} - \frac{9}{2} - 1$

$\frac{9}{4} - \frac{9}{4}$ ... $\frac{4}{4}$ ... $\frac{-13}{4} = -3.25$

$y = 2^2 + 3(2) - 1$

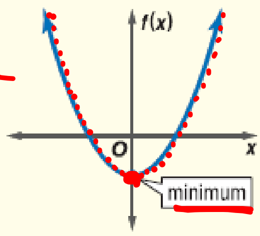
$y = 4 + 6 - 1$

$y = 9$

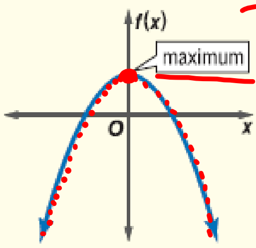
Key Concept — *Maximum and Minimum Value*

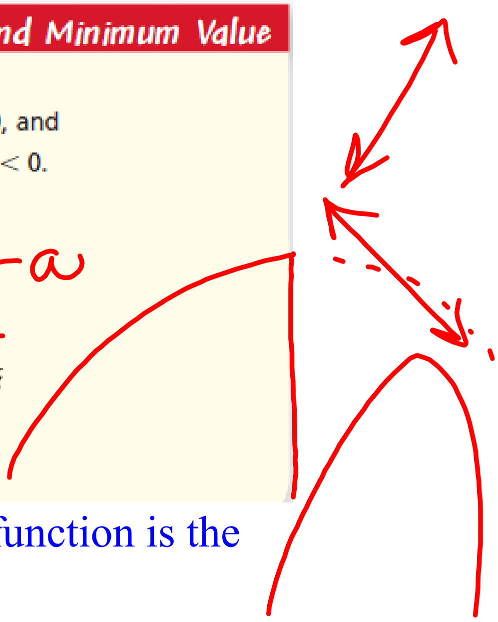
- **Words** The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,
  - opens up and has a minimum value when $a > 0$, and
  - opens down and has a maximum value when $a < 0$.
- **Models** a is positive. a is negative.

+a



-a





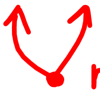
the ***maximum*** or ***minimum value*** of the function is the **y-coordinate** of the vertex.

- the x-coordinate *produces* the y-coordinate min. or max. value.

(t, h)

Example: **Determine whether the function has a minimum or maximum value. Then state it's maximum/minimum value.**

1. g(x) = x^2 - 4x + 9

$a=1$  min.

$x = \frac{-b}{2a}$

$x = \frac{4}{2(1)}$

$x = 2$

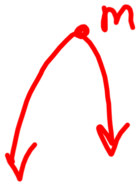
$y = 2^2 - 4(2) + 9$

$y = 4 - 8 + 9$

$y = 5$

$\boxed{\begin{array}{l} \text{min value} \\ @\,5 \\ d: \mathbb{R} \\ r: y \geq 5 \end{array}}$

2. h(x) = -x^2 + 7

$a = -1$  max

$x = \frac{-b}{2a}$

$x = 0$

$y = -(0)^2 + 7$

$y = 7$

max value @ 7

d: $\mathbb{R}$

r: $y \leq 7$

A souvenir shop sells about 200 coffee mugs each month for $6 each. The shop owner estimates that for each $.50 increase in the price, he will sell about 10 fewer coffee mugs per month.

a. how much should the owner charge for each mug in order to maximize the monthly income from their sales?

b. What is the maximum monthly income the owner can expect to make from the mugs?