

4-4 Imaginary & Complex Numbers

- simplify imaginary & complex numbers
- solve for an unknown using imaginary & complex numbers

imaginary unit = i i

$$i^2 = (-1)^2$$

$$i^2 = -1$$



definition of a pure imaginary number: for any positive number b ,

$$\sqrt{-b^2} = \sqrt{-1} \cdot \sqrt{b^2}$$

$$ib$$

$$bi$$

Examples: Simplify the following.

1. $\sqrt{-28}$

$$\cancel{\sqrt{-1}} \cdot \sqrt{28}$$

$$i \cancel{\sqrt{-1}} \cdot \sqrt{7}$$

$$2i\sqrt{7}$$

2. $\sqrt{-12} \cdot \sqrt{-2}$

$$i\sqrt{12} \cdot i\sqrt{2}$$

$$i^2 \sqrt{24}$$

$$\cancel{i^2} \cancel{\sqrt{-1}} \cdot \sqrt{6}$$

$$-1 \cdot 2\sqrt{6}$$

$$-2\sqrt{6}$$

3. $\sqrt{-32y^3}$

$$\cancel{i\sqrt{-1}} \cdot \sqrt{32y^3}$$

$$4y i \sqrt{2y}$$

4. $-3i \cdot 2i$

$$-6i^2$$

$$-6(-1)$$

$$6$$

Simplifying the Powers of i

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do you see a pattern?



i	i	i ⁵	i ² · i ² · i = (-1)(-1)i = i
i ²	-1	i ⁶	i ² · i ² · i ² = (-1)(-1)(-1) = -1
i ³	i ² · i = (-1)i = -i	i ⁷	i ² · i ² · i ² · i = -i
i ⁴	i ² · i ² = (-1)(-1) = 1	i ⁸	i ² · i ² · i ² · i ² = (-1)(-1)(-1)(-1) = 1

Pattern for simplifying the powers of i:

even exp. · 1 or -1
multiples of 4 = 1

odd exp. · i or -i
rewrite as previous even · i

Simplify the following powers of i.

1. i³⁵ i or -i

2. i¹⁰⁰ 1 or -1

3. i⁴⁶ 1 or -1

4. i⁴¹ i or -i (odd)

i³⁴ · i
-1(i) = -i

1

-1

i⁴⁰ · i
1(i) = i

Solve.

1. x² + 7 = 0

$\sqrt{x^2} = \sqrt{-7}$
x = ±i√7

$\sqrt{x^2} = \sqrt{6}$
x = ±4

2. a² + 72 = 0

$\sqrt{a^2} = \sqrt{-72}$
a = ±i√72
a = ±6i√2

~~√9√8~~
√4√2
3√8
2
6√2

Definition of a complex number: a number that can be written in the form $(a + bi)$ where a and b are **real numbers** and i is the **imaginary unit**.

- a is called the **real part**.
- b is called the **imaginary part**.
- two complex numbers are equal IFF the real parts are **equal** and the imaginary parts are **equal**. ★

To Simplify Imaginary & Complex Numbers

- NO imaginary roots in the radicand.
- NO imaginary/complex numbers in the denominator.
- Powers of i MUST be simplified. $i^4 = 1$
- Combine like terms and multiply i like all other variables.

$$\sqrt{-3} \rightarrow i\sqrt{3}$$



Simplify.

$$1. (8 + 7i) + (12 + 11i)$$

$$20 + 18i$$

$$2. (-7 + 4i) - (3 + 2i)$$

$$-10 + 2i$$

$$3. (8 + 5i)(2 - 3i)$$

$$16 - 24i + 10i - 15i^2$$

$$(31 - 14i)$$

$$4. (4 - 3i)(4 + 3i)$$

$$16 + 12i - 12i + 9$$

$$25$$

$$5. \frac{(3-9i)(4-2i)}{(4+2i)(4-2i)}$$

$$6. \frac{2+8i}{3i}$$

$$\frac{(12 - 6i - 36i - 18)}{16 - 8i - 8i + 4}$$

$$\frac{-6 - 42i}{20} = \frac{-3 - 21i}{10}$$

Solve for x and y.

7. $5x - 3yi = 2 + 9i$

8. $4x + 9i = 12 + y$

