4-5B Solving Quadratic Equations by Completing the Square

- *Solve quadratic equations using the square root property*
- *N.CN.7, F.IF.8a*

$\left(\frac{b}{2}\right)^2 = ac$

x^2+6x+9

$x^2+24x+144$

$9x^2+60x+100$

Completing the square: a process in making a **"non-perfect"** trinomial into a **perfect square trinomial**.

1. Get a = 1 *(divide every term by a)*
2. Isolate the variable. *(move c to the other side)*
3. Complete the square for the remaining terms. $(x^2 + bx + \_\,)$
   - add $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
4. Factor the now perfect square trinomial. $(x + \frac{b}{2})^2$
5. Solve using the square root property.

Determine the value of c that makes the following perfect square trinomials.

$\frac{-10}{2} = (-5)^2 = 25$

1. $x^2 - 10x + c$ → 25

$(x-5)^2$

$\frac{14}{2} = (7)^2 = 49$

2. $m^2 + 14m + c$ → 49

$(m+7)^2$

$\left(\frac{-9}{2}\right)^2 = \frac{81}{4}$

$\sqrt{\frac{81}{4}} = \frac{\sqrt{81}}{\sqrt{4}} \quad \frac{9}{2}$

3. $x^2 - 9x + c$ → $\frac{81}{4}$

$\left(x - \frac{9}{2}\right)^2$

1.44

1.6

$\sqrt{26.25} = 4.5$

$\sqrt{2025} = 45$

Examples

Solve the following quadratic equations by completing the square.

1. $x^2 + 6x = 16$

$\frac{6}{2} = (3)^2 = 9$

$x^2 + 6x + 9 = 16 + 9$

$\sqrt{(x+3)^2} = \sqrt{25}$

$x + 3 = \pm 5$

$-3 \quad -3$

$\{2, -8\}$

$x = -3 + 5 = 2$

$x = -3 - 5 = -8$

2. $x^2 - 4x + 1 = 0$

$-\frac{4}{2} = (-2)^2 = 4$

$x^2 - 4x + 4 = -1 + 4$

$\sqrt{(x-2)^2} = \sqrt{3}$

$x - 2 = \pm\sqrt{3}$

$x = 2 \pm \sqrt{3}$

3. $x^2 + 6x + 13 = 0$

$\frac{6}{2} = (3)^2 = 9$

$x^2 + 6x + 9 = -13 + 9$

$\sqrt{(x+3)^2} = \sqrt{-4}$

$x + 3 = \pm 2i$

$x = -3 \pm 2i$

4. $3x^2 + 4x = 8$

$\frac{3}{3} \quad \frac{4}{3} \quad \frac{8}{3}$

$\frac{4}{3} \cdot \frac{1}{2} = \frac{4}{6} \left(\frac{3}{3}\right)^2$

$\frac{4}{9}$

$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{24}{9} + \frac{4}{9}$

$\sqrt{\left(x + \frac{2}{3}\right)^2} = \sqrt{\frac{28}{9}} = \frac{\sqrt{28}}{\sqrt{9}}$

$x + \frac{2}{3} = \pm\frac{\sqrt{28}}{3}$

$x = -\frac{2}{3} \pm \frac{\sqrt{28}}{3}$

$x = \frac{-2 \pm \sqrt{28}}{3}$

$x = \frac{-2 \pm 2\sqrt{7}}{3}$

$17 \times 10 = 1.5 \times 10$

$\sqrt{3.25}$

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