4-6 The Quadratic Formula and the Discriminant

previous assignment: 4-5 WS

- *solve quadratic equations using the quadratic formula*
- *use the value of the discriminant to determine the number and nature of the roots.*
- *N.CN.7, A.SSE.1b*

erase

A formula that works for ANY quadratic equation

- *rational solutions*
- *irrational solutions*
- *imaginary solutions*

For any quadratic equation $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the following by using the quadratic formula.

$a=1$ $b=-10$ $c=24$

1. $t^2 - 10t + 24 = 0$

$(t-6)(t-4)=0$

$t-6=0$ or $t-4=0$

$\{6,4\}$

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$t = \frac{10 \pm \sqrt{(-10)^2 - 4(1)(24)}}{2(1)}$$

$t^2-10t+25=-24+25$

$\sqrt{(t-5)^2} = \sqrt{1}$

$t-5=\pm 1$

$5+1=6$

$t=5\pm 1$

$5-1=4$

$\{6,4\}$

$$t = \frac{10 \pm \sqrt{100-96}}{2}$$

$4=$ disc.

2 rational sol.

$$t = \frac{10 \pm \sqrt{4}}{2}$$

$$\frac{10+2}{2} = \frac{12}{2} = 6$$

$$t = \frac{10 \pm 2}{2}$$

$$\frac{10-2}{2} = \frac{8}{2} = 4$$

$\{4,6\}$

Solve the following by using the quadratic formula.

2. $\frac{3x^2 - 18x + 27}{3} = \frac{0}{3}$

$x^2 - 6x + 9 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$

$x = \frac{6 \pm \sqrt{36 - 36}}{2}$

$x = \frac{6 \pm \sqrt{0}}{2}$ → 0 = discriminant

1 rational

$x = \frac{6}{2}$ $\{3\}$

Solve the following by using the quadratic formula.

3. $5m^2 + 7m = -3$

$5m^2 + 7m + 3 = 0$

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$m = \frac{-7 \pm \sqrt{(7)^2 - 4(5)(3)}}{2(5)}$

$m = \frac{-7 \pm \sqrt{49 - 60}}{10}$

-11

2 complex sol.

$m = \frac{-7 \pm \sqrt{-11}}{10}$

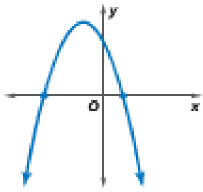
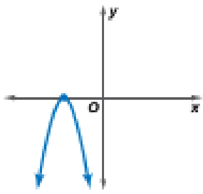
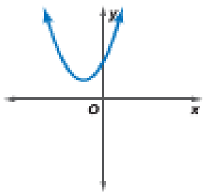
$m = \frac{-7 \pm i\sqrt{11}}{10}$

discriminant: the expression under the radical sign: $b^2 - 4ac$

- used to determine the ***nature*** of the roots. how many + what type

KeyConcept Discriminant

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

Use the discriminant to determine the nature of the roots.

1. $4x^2 = -25 + 20x$

$4x^2 - 20x + 25 = 0$

$b^2 - 4ac$

$(-20)^2 - 4(4)(25)$

$400 - 400$

$0 =$ Discr

1 rational sol.

2. $3x^2 + 2 = 5x$

$3x^2 - 5x + 2 = 0$

$b^2 - 4ac$

$(-5)^2 - 4(3)(2)$

$25 - 24$

1

2 rational sol.

ConceptSummary Solving Quadratic Equations

Method	Can be Used	When to Use
graphing	sometimes	Use only if an exact answer is not required. Best used to check the reasonableness of solutions found algebraically.
factoring	sometimes	Use if the constant term is 0 or if the factors are easily determined. **Example** $x^2 - 7x = 0$
Square Root Property	sometimes	Use for equations in which a perfect square is equal to a constant. **Example** $(x - 5)^2 = 18$
completing the square	always	Useful for equations of the form $x^2 + bx + c = 0$, where b is even. **Example** $x^2 + 6x - 14 = 0$
Quadratic Formula	always	Useful when other methods fail or are too tedious. **Example** $2.3x^2 - 1.8x + 9.7 = 0$