

## 5-3 Polynomial Functions

*Characteristics of a polynomial function in one variable:*

- the function is written with 1 variable.  ~~$-3x^2 + 5x$~~
- coefficients can be both real or imaginary.  $-3ix$
- exponents of the variables must be non-negative.  
 $x^{-3} = \frac{1}{x^3}$   $\sqrt{x} = x^{1/2}$  (no variable in denom.)

Examples: Are the following polynomial functions in 1 variable?

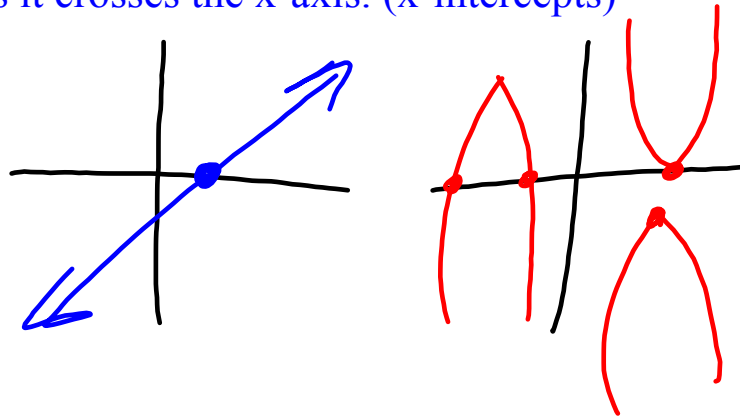
1.  $x^2 + 2xy + y^2$  no, more than 1 variable
2.  $2a^2 - 2a + .25$  yes
3.  $12 - \frac{2}{n} + n^2$  no, variable in the denom.

$$26. \quad \frac{2}{3} \Big| 2 \quad -1 \quad 3$$

**degree:** the largest exponent of it's variable.

**leading coefficient** coefficient of the term with the highest degree.

**real zeros:** the number of real zeros of a function is determine by the number of times it crosses the x-axis. (x-intercepts)



### Special Polynomial Functions

Degree	Name	Graph
$y = 3x^0$ 0	constant	
$y = 2x + 1$ 1	linear	
$y = -x^2$ 2	quadratic	
$y = x^3$ 3 $y = -x^3 - 2x^2 + 3$	cubic	
$y = x^4$ 4 $y = -x^4 - 1$	quartic	
5 $y = x^5$	quintic	

**Examples:** Find the degree of each polynomial expression, then state the lead coefficient for each.

1.  $-3x^5 + 4x^2 - 7$

degree: **5**

lead coefficient: **-3**

2.  $2x^2 + 5x^3 + 6$

degree: **3**

lead coefficient: **5**

3.  $x + 1$

degree: **1**

lead coefficient: **1**

4.  $-x^6 - 3$

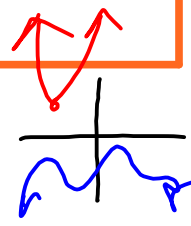
degree: **6**

lead coefficient: **-1**

Graph	Degree	Lead Coefficient	End Behavior
	even	positive	$f(x) \Rightarrow +\infty$ as $x \Rightarrow -\infty$ . $f(x) \Rightarrow +\infty$ as $x \Rightarrow +\infty$ .
	even	negative	$f(x) \Rightarrow -\infty$ as $x \Rightarrow -\infty$ . $f(x) \Rightarrow -\infty$ as $x \Rightarrow +\infty$ .

**Even Degree Graph Summary:**

- even degree polynomials may not cross the x-axis.
- even degree, positive lead: starts up, ends up.
- even degree, negative lead: starts down, ends down.

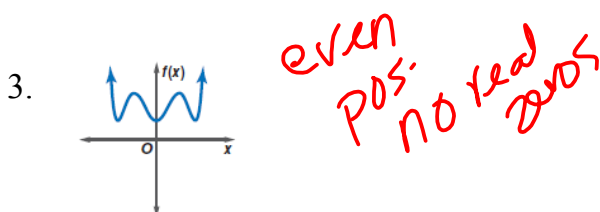
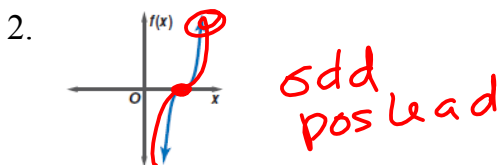
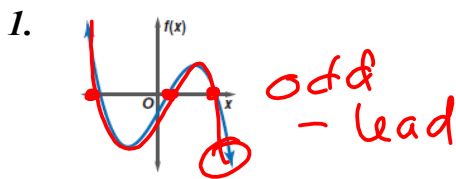


Graph	Degree	Lead Coefficient	End Behavior
	odd	positive	$f(x) \rightarrow -\infty$ as $x \rightarrow -\infty$ $f(x) \rightarrow +\infty$ as $x \rightarrow +\infty$
	odd	negative	$f(x) \rightarrow +\infty$ as $x \rightarrow -\infty$ $f(x) \rightarrow -\infty$ as $x \rightarrow +\infty$

**Odd Degree Graph Summary:**

- odd degree polynomials must cross the x-axis at least once.
- odd degree, positive lead: starts down, ends up.
- odd degree, negative lead: starts up, ends down.

**Examples:** For each graph, describe the end behavior, determine whether it represents an odd or even degree polynomial function, the sign of the lead coefficient and the number of real zeros.

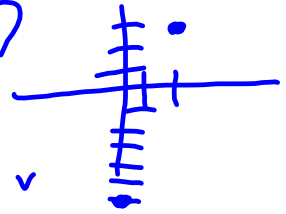


Evaluating a polynomial function for a given value.

1. find  $f(2)$  for  $f(x) = x^3 - 2x^2 + 5x - 7$

$$f(2) = (2)^3 - 2(2)^2 + 5(2) - 7$$
$$= 8 - 8 + 10 - 7$$

$$\begin{array}{r} x \\ (2 \overline{) 3} \end{array}$$



2. find  $g(-3)$  for  $g(x) = -x^3 - 5x^2 + 1$

$$g(-3) = -1 \cdot (-3)^3 - 5(-3)^2 + 1$$
$$= -81 - 45 + 1$$

$$g(-3) = -125$$

$$g(x+1) = -(x+1)^3 - 5(x+1)^2 + 1$$

p. 326

14-34 evens

35-40 all

incl +/- bad

