5-5 Solving Equations using Quadratic Techniques

We will use quadratic factoring techniques to locate zeros of ***non-quadratic*** functions.

- GCF
- differences of squares
- **sums and differences of perfect cubes**
- quadratic trinomials

$x^2(x^4 \quad)$

$x^2(\quad x \quad)()$

$(x \quad)(x \quad)(x \quad)$ $)(x)(x^8$

x^6

(x^2+3)

$(a^2-b^2)=(a+b)(a-b)$

$(a^3+b^3)=(\quad)(\quad)$

$(a^3-b^3)=(\quad)(\quad)$

Examples: ***Solve the following equations using quadratic techniques.***

1. $m^4 - 625 = 0$

2. $x^3 - 12x^2 + 20x = 0$

Factoring Sums and Differences of Perfect Cubes

1st 10 perfect cubes:
variable perfect cubes:

Sum of Perfect Cubes:

Differece of Perfect Cubes:

1. $(x^3 - 27)$

2. $(m^6 + 125)$

3. $(8n^3 + 64)$

4. $(27x^9 - 216)$

"Quadratic" Trinomials: $ax^2 + bx + c$

- the linear terms exponent must be 1/2 the quadratic terms exponent.
- used to rewrite non-quadratic trinomials as a quadratic trinomial.

Examples: ***Rewrite each of the following as a quadratic equation if possible.***

1. $x^4 - 7x^2 + 12 = 0$ $(x^2)^2 - 7(x^2)^1 + 12 = 0$

2. $x^{2/3} - 8x^{1/3} + 15 = 0$ $(x^{1/3})^2 - 8(x^{1/3}) + 15 = 0$

3. $x^4 + 3x^3 - 18 = 0$ not possible

Solving Quadratic Trinomials:

- rewrite $(x^2)^2 + 5(x^2) + 6$ let $m = x^2$
- substitute new variable $m^2 + 5m + 6$
- solve for new variable $(m+2)(m+3)$
- replace original variable
- finishing solving for original variable

Examples: Solve.

1. $x^4 - 13x^2 + 36 = 0$

$(x^2)^2 - 13(x^2) + 36 = 0$ let $m = x^2$

$m^2 - 13m + 36 = 0$

$(m-4)(m-9) = 0$

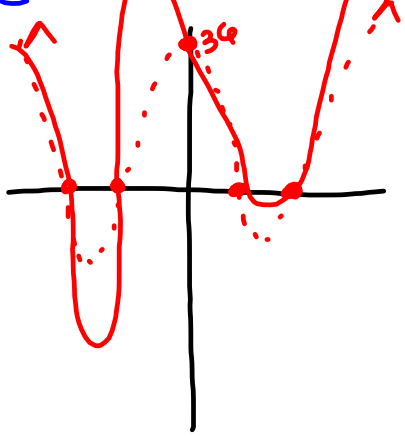
$m - 4 = 0$ or $m - 9 = 0$

$m = 4$ $\quad$ $m = 9$

$\sqrt{x^2} = \sqrt{4}$ $\quad$ $\sqrt{x^2} = \sqrt{9}$

$x = \pm 2$ $\quad$ $x = \pm 3$

$\{\pm 2, \pm 3\}$



$x^4 - 19x^2 + 48 = 0$

$(x^2)^2 - 19(x^2) + 48 = 0$ let $m = x^2$

$m^2 - 19m + 48 = 0$

$(m-3)(m-16) = 0$

$m - 3 = 0$ or $m - 16 = 0$

$m = 3$ $\quad$ $m = 16$

$\sqrt{x^2} = \sqrt{3}$ $\quad$ $\sqrt{x^2} = \sqrt{16}$

$x = \pm\sqrt{3}$ $\quad$ $x = \pm 4$

$\{\pm 4, \pm\sqrt{3}\}$

$x^3 + 27 = 0$

$(x+3)(x^2-3x+9) = 0$

$x+3=0$ or $x^2-3x+9=0$

$x=-3$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{3 \pm \sqrt{(-3)^2-4(1)(9)}}{2(1)}$

$x = \frac{3 \pm \sqrt{9-36}}{2}$

$x = \frac{3 \pm \sqrt{-27}}{2}$

$\sqrt{-27}$

$i\sqrt{27}$

$i\sqrt{9}\cdot\sqrt{3}$

$3i\sqrt{3}$

Factor.

( )

Rewrite

(x^2) solve

$x = \frac{3 \pm 3i\sqrt{3}}{2}$

$\left\{-3, \frac{3 \pm 3i\sqrt{3}}{2}\right\}$

$x^4 - 19x^2 + 48 = 0$

$(x^2)^2 - 19(x^2) + 48 = 0$ Let $m = x^2$

$m^2 - 19m + 48 = 0$

$(m-3)(m-16) = 0$

$m-3=0$ $m-16=0$

$m=3$ $m=16$

$\sqrt{x^2} = \sqrt{3}$ $\sqrt{x^2} = \sqrt{16}$

$x = \pm\sqrt{3}$ $x = \pm 4$

$\{\pm 4, \pm\sqrt{3}\}$

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