5-6 The Remainder & Factor Theorems

Review: ***Use long division and synthetic division to find the quotient of $-16x^2 + 80x + 5$ and $(x - 3)$.***

Long Division	**Synthetic Division**
$(x-r)$ $-16x + 32 + \frac{101}{x-3}$ $x-3 \overline{)\,-16x^2 + 80x + 5}$ $-\;\; -16x^2 + 48x$ $32x + 5$ $-\;\; 32x - 96$ 101	$3 \mid -16 \quad 80 \quad 5$ $\downarrow \quad -48 \quad 96$ $\cancel{-16 \quad 32} \mid 101 = f(3)$ $\boxed{\cancel{-16x + 32 + \frac{101}{x-3}}}$

Now find $f(3)$ for $f(x) = -16x^2 + 80x + 5$ $(x-3)$

$f(3) = -16(3)^2 + 80(3) + 5$

$f(3) = 101$

The Remainder Theorem: If a polynomial f(x) is divided by a binomial (x - a), then the remainder is the constant f(a).

$a \mid$ ______ $\mid \# = f(a)$

Synthetic Substitution: The process of using synthetic division to find the value of a function.

Examples: ***Use synthetic substitution to find the value of the following functions.***

1. Find f(10) when $f(x) = x^4 - 10x^3 + x^2 - 8x + 1$

$(x-10)$ $\quad (10)^4 - (10)^3 + (10)^2 - 8(10) + 1$

$10 \mid 1 \quad -10 \quad 1 \quad -8 \quad 1$
$\downarrow \quad 10 \quad 0 \quad 10 \quad 20$
$\cancel{1 \quad 0 \quad 1 \quad 2} \mid 21$

$\boxed{f(10) = 21}$

2. Find g(-2) when $g(x) = 3x^5 - 5x^3 + 57$

$\boxed{(x+2)}$

$-2 \mid 3 \quad 0 \quad -5 \quad 0 \quad 0 \quad 57$
$\downarrow \quad -6 \quad 12 \quad -14 \quad 28 \quad -56$
$3 \quad -6 \quad 7 \quad -14 \quad 28 \mid 1$

$\boxed{g(-2) = 1}$

$12 \overline{)36}$ = 3, -36, 0 $\quad$ $6 \overline{)36}$ = 6, -36, 0 $\quad$ 6 $\quad$ $\cancel{3}$ $\quad$ 12 $\quad$ $\cancel{3}$ 9

The Factor Theorem: The binomial (x - a) is a factor of the polynomial f(x) IFF f(a) = 0.

a|

remaing factor

Depressed Polynomial: the remaining polynomial after a binomial is factored out of the original polynomial.

- depressed polynomials are 1 degree less than the previous polynomial.

- some depressed polynomials can be factored again.

deg 5

?| $\to$ deg 4

?| $\to$ deg 3

$\to$ deg 2

ax^2+bx+c

( )( )

Examples: ***Given a polynomial and one of its factors, find the remaining factors of the polynomial.***

1. $x^3 + 7x^2 + 2x - 40$; (x - 2)

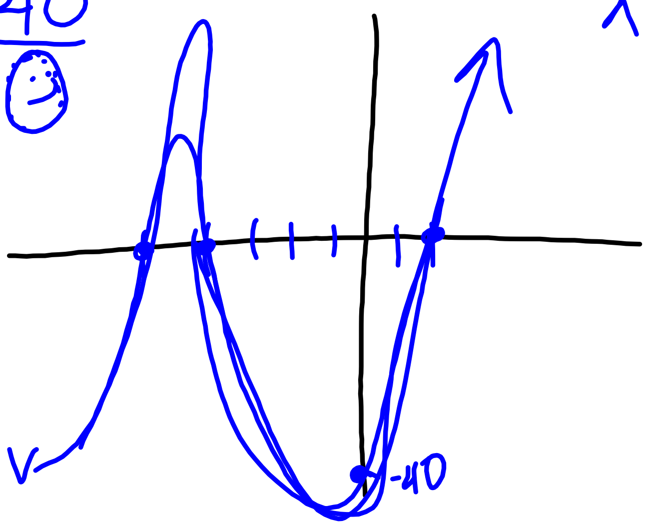
$(x-2)\boxed{(x^2 \quad . \quad . \quad)}$

$(x-2)\boxed{(x+4)(x+5)}$

$$\begin{array}{r|rrrr} 2 & 1 & 7 & 2 & -40 \\ & \downarrow & 2 & 18 & 40 \\ \hline & 1 & 9 & 20 & 0 \end{array}$$

$x^2+9x+20$

$(x+4)(x+5)$



Examples: ***Given a polynomial and one of its factors, find the remaining factors of the polynomial.***

2. $6x^3 - 25x^2 + 2x + 8$; $(3x - 2)$

$\frac{6x^3 - 25x^2 + 2x + 8}{3}$; $\frac{(3x-2)}{3}$

$(x - \frac{2}{3})$

$\cancel{(3x-2)(2x^2 \quad)}$

$\boxed{(3x-2)(2x+1)(x-4)}$

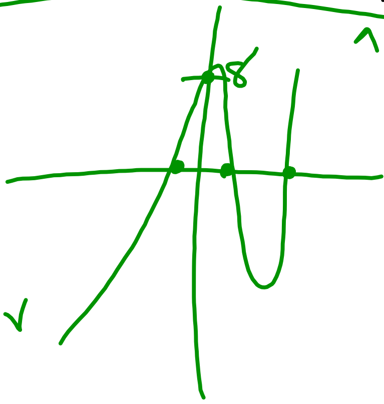
$\frac{2}{3}$	2	$-\frac{25}{3}$	$\frac{2}{3}$	$\frac{8}{3}$
	$\downarrow$	$\frac{4}{3}$	$-\frac{14}{3}$	$-\frac{8}{3}$
	2	-7	-4	☺

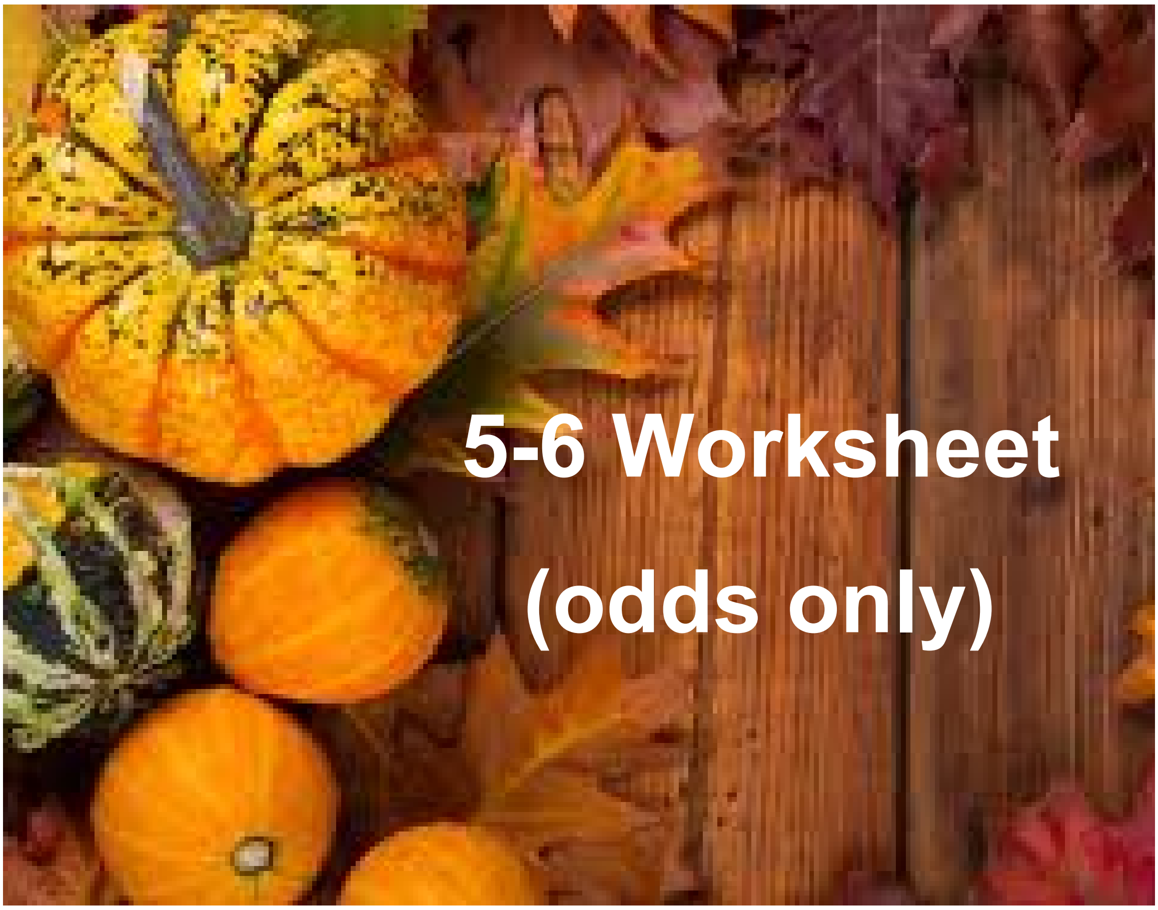
$2x^2 - 7x - 4$

$\boxed{(2x+1)(x-4)}$

x	+
-8	-7

-8 1





-1 | $1 \quad 1 \quad -5 \quad -5 \quad 4 \quad 4$ (deg 5)

↓ $-1 \quad 0 \quad 5 \quad 0 \quad -4$

2 | $1 \quad 0 \quad -5 \quad 0 \quad 4$ | 0 (deg 4)

↓ $2 \quad 4 \quad -2 \quad -4$

? | $1 \quad 2 \quad -1 \quad -2$ | 0 (deg 3)

↓

| 0 (deg 2)

$x^2 + x + c$

$(\; x \quad)$