5-7 Roots and Zeros

Summary of Factors, Roots, and Zeros

$(x-3)$

$f(3)=0$ 3

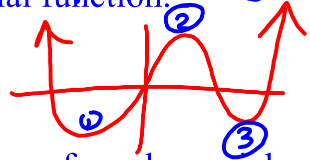
For any function f(x), if f(a) = 0, then

$\{3\}$ (3, 0)

- (x - a) is a factor of the polynomial.
- **a** is called a **solution**, **root**, or **zero** of the polynomial function.
- If **a** is a real number, then (a, 0) is an x-intercept of the graph of the function.

Total number of zeros = degree of the polynomial function.

deg=4



Total number of direction changes = degree - 1

$a \pm bi$

Complex Conjugates Theorem: If $a + bi$ is a zero of a polynomial function, then $a - bi$ is also a zero of the polynomial function.

$2+3i \rightarrow 2-3i \quad 3i \Rightarrow -3i$

imag. come in pairs

even amt.

Descartes' Rule of Signs: If f(x) is a polynomial function with real coefficients whose terms are arranged in ***descending*** powers of the variable, then:

$x^4 \quad x^3 \; . \; . \; . \; -3$

- the number of ***positive real zeros*** of the function is the same as the number of changes in sign of the coefficients of the terms of **f(x)**, or less this amount by an even number.
- the number of ***negative real zeros*** of the function is the same as the number of changes in sign of the coefficients of the terms of **f(-x)**, or less this amount by an even number.
- the number of ***imaginary zeros*** of the function is determined by considering the combinations of total zeros, positive zeros, and negative zeros.

Fill in gaps

even pairs

Examples: ***State the possible number of positive real zeros, negative real zeros, and imaginary zeros for the following polynomial functions.***

1. $p(x) = x^5 - 6x^4 - 3x^3 + 7x^2 - 8x + 1$

pos: $+ \; - \; - \; + \; - \; +$ $\boxed{\text{4 or 2 or 0 pos.}}$

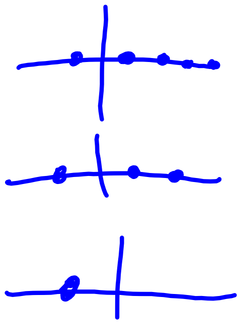
neg: $p(-x) = (-x)^5 - 6(-x)^4 - 3(-x)^3 + 7(-x)^2 - 8(-x) + 1$

$- \; - \; + \; + \; + \; +$

$\boxed{\text{1 neg}}$

5 total zeros

+ + + + −	0 Imag.
+ + − i i	2 Imag.
− i i i i	4 Imag.



Examples: ***State the possible number of positive real zeros, negative real zeros, and imaginary zeros for the following polynomial functions.***

2. $g(x) = -x^6 + 4x^3 - 2x^2 - x - 1$

pos: $- \; + \; - \; - \; -$ $\boxed{\text{2 or 0 pos.}}$

neg: $- \; - \; - \; + \; -$ $\boxed{\text{2 or 0 neg.}}$

6 total

+ + − − i i

+ + i i i i

− − i i i i

i i i i i i

$\boxed{\text{2, 4, or 6 imag.}}$

Examples: ***Find ALL the zeros of the following polynomial functions.***

3. $f(x) = x^3 - x^2 + 2x + 4$

pos: + − + + : 2 or 0 pos.

neg: − − − + : 1 neg.

−1	1	−1	2	4
	↓	−1	2	−4
	1	−2	4	0

$x^2 - 2x + 4 = 0$

use quad. formula to solve.

{ −1 }

_ , _ , _

3 total

\+ \+ −

− i i

2 or 0 imag.

Examples: ***Find ALL the zeros of the following polynomial functions.***

4. $f(x) = x^3 - 4x^2 + 6x - 4$

Example: Given the solutions, write the polynomial function.

5. $\{2, -3, 4i, -4i\}$

$\boxed{f(x) = x^4 \; . \; . \; . \; . \; . \; . \; . \; .}$

$x = 2 \qquad x = -3 \qquad x = 4i \qquad x = -4i$

$f(x) = \boxed{(x-2)(x+3)}\boxed{(x-4i)(x+4i)}$