5-8 Rational Zero Theorem ~~3-5~~ 5.3

Rational Zero Theorem: Let f(x) represent a polynomial function with a_0 as the lead coefficient and a_n as its constant. If $\frac{p}{q}$ is a rational number in simplest form and is a zero of function f(x), then p is a factor of a_n and q is a factor of a_0.

constant lead

Example: Let $f(x) = 2x^3 + 3x^2 - 17x + 12$

a_0 a_n

p (factors of the constant): (12) $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

q (factors of the lead coefficient): (2): ±1, ±2

$\frac{p}{q}$ ***(possible rational zeros of f(x):*** $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12,$
$\pm\frac{1}{2}, \pm\frac{3}{2}$

Examples: ***List all the possible rational zeros of each function.***

1. f(x) $= 3x^4 - x^3 + 4$

p: (4): $\pm1, \pm2, \pm4$

q: (3): $\pm1, \pm3$

$\frac{p}{q}$: $\pm1, \pm2, \pm4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

2. $h(x) = x^4 + 7x^3 - 15$

p: (15) $\pm1, \pm3, \pm5, \pm15$ $\frac{p}{q}$

q: ±1

Examples: *Find all the zeros of the following function.*

3. $f(x) = 2x^4 - 13x^3 + 23x^2 - 52x + 60$

total zeros: 4

positive zeros: + − + − + : 4, 2, or 0 positive

negative zeros: + + + + + : 0 neg.

imaginary zeros: 0, 2, 4 imag.

combinations of zeros (4)

+ + + +

+ + i i

i i i i

$\{5, \tfrac{3}{2}, \pm 2i\}$

$p =$ 60 : 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

$q =$ 2 : 1, 2

$\frac{p}{q} =$ $\cancel{1}, \cancel{2}, \cancel{3}, \cancel{4}$, 5, $\cancel{6}, \cancel{10}, \cancel{12}, \cancel{15}, \cancel{20}, \cancel{30}, \cancel{60}, \cancel{\tfrac{1}{2}}$, $\tfrac{3}{2}$, $\tfrac{5}{2}$, $\tfrac{15}{2}$

$$
\begin{array}{r|rrrrr}
5 & 2 & -13 & 23 & -52 & 60 \\
 & \downarrow & 10 & -15 & 40 & -60 \\
\hline
\tfrac{3}{2} & 2 & -3 & 8 & -12 & 0 \ (\text{deg } 3) \\
 & \downarrow & 3 & 0 & 12 & \\
\hline
 & 2 & 0 & 8 & 0 & \text{deg } 2
\end{array}
$$

$2x^2 + 8 = 0$

$2x^2 = -8$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm 2i$