6-5 Radical Expressions

- *simplify radicals*
- *add or subtract radicals*
- *multiply and divide radicals*

A radical expression is in simplified form when the following conditions are met:

- the radicand contains no perfect factors.
- the radicand contains no fractions.
- no radicals are in the denominator.
- the index is as small as possible. (section 6-6)

x^2: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144

x^3: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

x^4: 1, 16, 81, 256, 625, 1296

x^5: 1, 32, 243, 1024

x^6: 1, 64,

Product Property of Radicals

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$

~~If n is even, both a and b must be positive to use the product property!~~

$\sqrt{20}$

$\sqrt{4} \cdot \sqrt{5}$

$2\sqrt{5}$

Quotient Property of Radicals

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$\sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{9}$

$\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$

Examples:

Simplify.

1. $\sqrt{24a^3b^2}$

$\cancel{\sqrt{4}} \cdot \sqrt{6}\, \cancel{\sqrt{a^2}} \sqrt{a}\, \cancel{\sqrt{b^2}}$

$2a|b|\sqrt{6a}$

2. $\sqrt[2]{64a^2b^3c^1}$

$2\overline{)3}$
-2
1

$\cancel{\sqrt{64}}\, \cancel{\sqrt{a^2}}\, \cancel{\sqrt{b^2}} \sqrt{b} \sqrt{c}$

$8|a|b^1 \sqrt{b^1c}$

3. $\sqrt[3]{54a^3b^7c^{11}}$

$\sqrt[3]{\cancel{27}} \cdot \sqrt[3]{2}\ \sqrt[3]{a^3b^7}$

$3ab^2c^3\sqrt[3]{2bc^2}$

4. $\sqrt{10x^2y} \cdot \sqrt{40xy^3}$

$\sqrt[2]{400x^3y^4}$

$20xy^2\sqrt{x}$

Simplify.

5. $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{\cancel{\sqrt{4}}\,2} = \frac{\sqrt{5}}{2}$

6. $\sqrt[3]{\frac{3}{8}} = \frac{\sqrt[3]{3}}{\cancel{\sqrt[3]{8}}\,2} = \frac{\sqrt[3]{3}}{2}$

7. $\frac{20\sqrt{8}}{2\sqrt{2}} = \frac{10\sqrt{8}}{\sqrt{2}} = \cancel{\frac{10\sqrt{\cancel{4}}^{\,2}\sqrt{2}}{\sqrt{2}}} = 20$

$10\sqrt{\frac{8}{2}} = 10\sqrt{\cancel{4}}_{\,2} = 20$

$\frac{10\sqrt{8}\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{10\cancel{\sqrt{64}}\,4}{\cancel{\sqrt{4}}\,2} = \frac{40}{2} = 20$

Multiplying Rational Numbers and Radicals

- only ***common roots*** can be multiplied together *(same indices)*
- multiply the outside coefficients, multiply the inside radicands.

Simplify.

8. $3\sqrt{2}\cdot 4\sqrt{10}$

$12\sqrt{20}$

$12\sqrt{5}\sqrt{\cancel{4}}$ 2

$24\sqrt{5}$

9. $4\sqrt{3}(\sqrt{27}+4\sqrt{3})$

$4\sqrt{\cancel{81}}$ 9 $+16\sqrt{\cancel{9}}$ 3

$36+48$

84

Rationalizing the Denominator: method for eliminating radicals in the denominator.

- multiply the numerator and denominator by a quantity to give a perfect root in the denominator.

Simplify.

10. $\dfrac{5}{2\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{5\sqrt{3}}{2\sqrt{\cancel{9}}\ 3} = \boxed{\dfrac{5\sqrt{3}}{6}}$

11. $\sqrt{\dfrac{4}{ab^2}} = \dfrac{\sqrt{4}}{\sqrt{ab^2}} = \dfrac{2}{b\sqrt{a}}\cdot\dfrac{\sqrt{a}}{\sqrt{a}} = \dfrac{2\sqrt{a}}{b\sqrt{a^2}} = \boxed{\dfrac{2\sqrt{a}}{ab}}$

12. $\sqrt[3]{\dfrac{2}{3t}}$ $\quad \dfrac{\sqrt[3]{2}}{\sqrt[3]{3t}}\cdot\dfrac{\sqrt[3]{9t^2}}{\sqrt[3]{9t^2}} = \dfrac{\sqrt[3]{18t^2}}{\cancel{\sqrt[3]{27t^3}}\ 3t}$ $\quad \dfrac{\sqrt[3]{18t^2}}{3t}$

Adding & Subtracting Radicals: combine "like" radicals:

  * same index

  * same radicand

- add or subtract the coefficients in front of the radical.
- **radicand and index REMAIN THE SAME!**

Simplify.

13. $2\sqrt{3} + 5\sqrt{2} + 7\sqrt{3} - \sqrt{2} = \boxed{9\sqrt{3} + 4\sqrt{2}}$

14. $4\sqrt{27} + 3\sqrt{3} - \sqrt{48}$

$4\sqrt{9}\sqrt{3} + 3\sqrt{3} - \sqrt{16}\sqrt{3}$

$12\sqrt{3} + 3\sqrt{3} - 4\sqrt{3} = 11\sqrt{3}$

15. $(2\sqrt{5} + 3\sqrt{2})(4\sqrt{5} - \sqrt{2})$

FOIL

16. $(5 + \sqrt{3})(5 - \sqrt{3})$

Conjugates: binomials of the form $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$

- use conjugates to rationalize a binomial denominator containing a radical.
- multiply numerator and denominator by the conjugate.

Simplify.

18. $\dfrac{4}{5 - \sqrt{3}} \cdot \dfrac{(5+\sqrt{3})}{(5+\sqrt{3})} = \dfrac{20 + 4\sqrt{3}}{25 - \cancel{\sqrt{9}}\,3} = \dfrac{20+4\sqrt{3}}{22}$

$\dfrac{10 + 2\sqrt{3}}{11}$

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(skip 36)