7-2A Solving Exponential Equations & Inequalities

Properties of Exponents Review

- $a^m(a^n) = a^{m+n}$

- $(a^m)^n = a^{mn}$

- $a^m(a)^{-n} = \dfrac{a^m}{a^n} = a^{m-n}$

Solving Exponential Equations

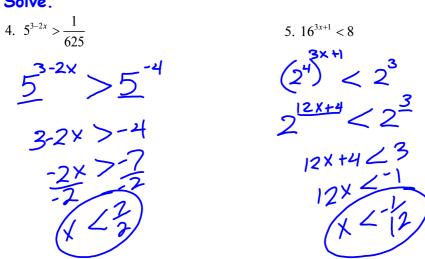**Exponential Equation:** an equation where the variable occurs as the exponent.

KeyConcept Property of Equality for Exponential Functions

Words	Let $b > 0$ and $b \neq 1$. Then $b^x = b^y$ if and only if $x = y$.
Example	If $3^x = 3^5$ then $x = 5$

Solve.

$3^{-10} \overset{?}{=} 9^{-5}$

1. $4^{9n-2} = 256$

$4^{9n-2} = 4^4$

$9n - 2 = 4$

$9n = 6$

$n = \dfrac{6}{9}$

$n = \dfrac{2}{3}$

2. $3^{5x} = 9^{2x-1}$

$3^{5x} = (3^2)^{2x-1}$

$3^{5x} = 3^{4x-2}$

$5x = 4x - 2$

$x = -2$

3. $\left(\dfrac{1}{9}\right)^{3c+1} = 27^{3c-1}$

$\left(\dfrac{1}{3^2}\right)^{3c+1} = (3^3)^{3c-1}$

$(3^{-2})^{3c+1} = (3^3)^{3c-1}$

$3^{-6c-2} = 3^{9c-3}$

$-6c - 2 = 9c - 3$

$1 = 15c$

$\dfrac{1}{15} = c$

Solving Exponential Inequalities

Exponential Inequality: an inequality involving **exponential functions.**

KeyConcept Property of Inequality for Exponential Functions

Words	Let $b > 1$. Then $b^x > b^y$ if and only if $x > y$, and $b^x < b^y$ if and only if $x < y$.
Example	If $2^x > 2^6$, then $x > 6$. W

Solve.

4. $5^{3-2x} > \dfrac{1}{625}$

$$5^{3-2x} > 5^{-4}$$

$$3-2x > -4$$

$$\dfrac{-2x}{-2} > \dfrac{-7}{-2}$$

$$\boxed{x < \dfrac{7}{2}}$$

5. $16^{3x+1} < 8$

$$\left(2^4\right)^{3x+1} < 2^3$$

$$2^{12x+4} < 2^3$$

$$12x + 4 < 3$$

$$12x < -1$$

$$\boxed{x < \dfrac{-1}{12}}$$

Compound Interest: when interest is paid on the **principal** of an investment **AND** any previous earned **interest.**

KeyConcept Compound Interest

You can calculate compound interest using the following formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt},$$

where A is the amount in the account after t years, P is the principal amount invested, r is the annual interest rate, and n is the number of compounding periods each year.

annually, $n = 1$ monthly $= 12$

semi-annual, $n = 2$ daily $= 365$

quarterly $4 = n$

Solve.

6. An investment account pays 5.4% annual interest compounded quarterly. If $4,000 is deposited into the account initially, what is the balance of the account after 8 years?

$$y = 4{,}000\left(1 + \frac{.054}{4}\right)^{8(4)}$$

$$y = 4000(1.0135)^{32}$$

$$y = 4000(1.5358)$$

$$\boxed{y = \$6{,}143.56}$$

P. 464, 465

10 - 14e
20 - 28e
32 - 36e