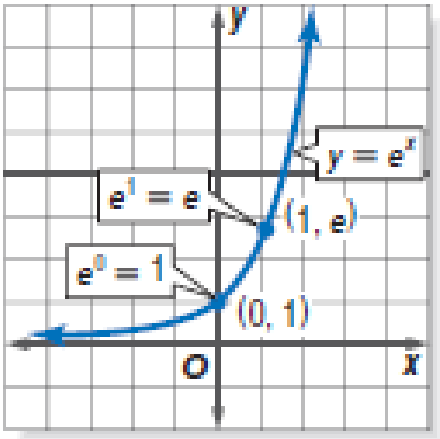
7-7 Base e and Natural Logarithms

natural base e $\quad y = e^x \quad e \approx 2.71828$

- finance - continuously compounded interest.
- science - continuous growth/decay
- all exponential properties apply to base ***e***.

how often compounded	computation
yearly	$\left(1+\frac{1}{1}\right)^1 = 2$
semi-annually	$\left(1+\frac{1}{2}\right)^2 = 2.25$
quarterly	$\left(1+\frac{1}{4}\right)^4 = 2.44140625$
monthly	$\left(1+\frac{1}{12}\right)^{12} \approx 2.61303529022\ldots$
weekly	$\left(1+\frac{1}{52}\right)^{52} \approx 2.69259695444\ldots$
daily	$\left(1+\frac{1}{365}\right)^{365} \approx 2.71456748202\ldots$
hourly	$\left(1+\frac{1}{8760}\right)^{8760} \approx 2.71812669063\ldots$
every minute	$\left(1+\frac{1}{525600}\right)^{525600} \approx 2.7182792154$
every second	$\left(1+\frac{1}{31536000}\right)^{31536000} \approx 2.71828247254\ldots$



51. $\dfrac{p \cdot \log 6}{\log 13} \le \dfrac{(5-p)\log 13}{\log 13}$

$\dfrac{0.6986p}{1+p} \le \dfrac{5-1p}{+p}$

$1.6986p \le 5$

$2p = 5 - 1p$

$+p \qquad +p$

$\dfrac{3p}{3} = \dfrac{5}{3}$

$p = \dfrac{5}{3}$

31. $\log 3^{y-1} \le \log 4^{y}$

$$\frac{(y-1)\cdot\cancel{\log 3}}{\cancel{\log 3}} \le \frac{y\cdot\log 4}{\log 3}$$

$$y-1 \le 1.2619y$$

Natural logarithm (ln): the logarithm of base e.

$$\log_e x = \ln x$$

Evaluate the natural base e expressions.

1. e^{2} 2. $e^{-1.3}$ 3. $e^{0.5}$ 4. e^{-8}

7.3891 0.2725 1.6487 0.0003

Evaluate the natural logarithm expressions.

5. $\ln 3$ 6. $\ln \frac{1}{4}$ 7. $\ln 0.05$ 8. $\ln 2.3$

$e^{?}=3$ 1.0986 -1.3863 -2.9957 0.8329

Since the natural base e function and the natural logarithmic function are **inverses**, they "**undo**" each other.

$\log_5 5^3$

Therefore: $\cancel{e^{\ln x}} = x$ **and** $\cancel{\ln e^x} = x$

$5^{\log_5 2}$

Evaluate using the inverse properties.

1. $\cancel{e^{\ln 3}}$

3

2. $\ln e^{2x-1}$

$2x-1$

3. $\cancel{e^{\ln 21}}$

21

$\cancel{\ln 3 =}$

Solving equations and inequalities using natural base e and natural logarithms

- when given $e^x = y$, take the $\ln e^x = \ln y$ *(take the ln of each side)*
- when given $\ln x = y$, then $e^{\ln x} = e^y$

$2 \cdot x = 3 \cdot 2$

$2x = 6$

Solve the following equations.

1. $3e^{-2x} + 4 = 10$

$3e^{-2x} = 6$

$\ln e^{-2x} = \ln 2$

$\frac{-2x}{-2} = \frac{\ln 2}{-2}$

$x \approx -0.3466$

2. $\ln(3x) = 0.5$

$e^{\ln(3x)} = e^{0.5}$

$\frac{3x}{3} = \frac{e^{0.5}}{3}$

$x \approx 0.5496$

3. $\ln(x) + \ln(3) = 5$

$e^{\ln 3x} = e^5$

$\frac{3x}{3} = \frac{e^5}{3}$

$x = 49.4711$

Solve the following inequalities.

$2x-3>0$

3. ln (2x - 3) < 2.5

$e^{\ln(2x-3)} < e^{2.5}$

$2x>3$

$x>\frac{3}{2}$

$2x-3<e^{2.5}$

$\frac{2x}{2}<\frac{e^{2.5}+3}{2}$

$x<7.5912$

$1.5<x$

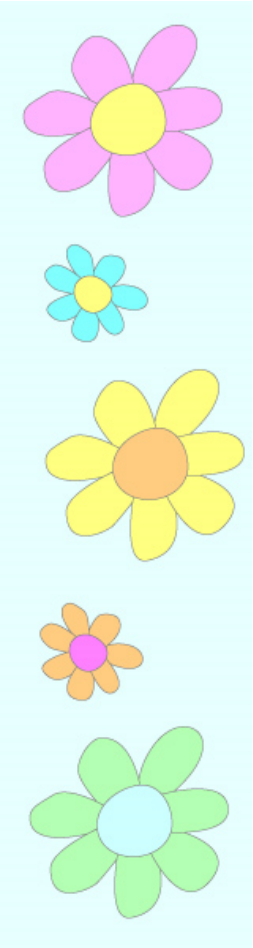
4. $-3e^x + 10 < 8$

$-3e^x<-2$

$e^x>\frac{2}{3}$

$\ln e^x>\ln\frac{2}{3}$

$x>-0.4055$



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