

8-1 Simplifying Rational Expressions

- *simplify rational expressions*
- *A.REI.*

rational expression: a quotient of two polynomials.

- fractions with polynomials as numerators and denominators
- the properties of fractions are used to simplify rational expressions.

undefined values: values of the variable or variables that make the denominator of the original expression equal to zero.

Simplify.

$$1. \frac{12}{45} \stackrel{\div 3}{=} \frac{4}{15}$$

~~$\frac{3 \cdot 2 \cdot 2}{5 \cdot 3 \cdot 3}$~~ $\left(\frac{4}{15}\right)$

Multiply.

$$2. \frac{3}{10} \cdot \frac{5}{9} \stackrel{\substack{\text{num} \\ \text{den}}}{=} \frac{15}{90} = \frac{1}{6}$$

~~$\frac{3 \cdot 5}{2 \cdot 5 \cdot 3 \cdot 3}$~~ $\left(\frac{1}{6}\right)$

Divide.

$$3. \frac{4}{15} \div \frac{2}{9}$$

$\frac{4}{15} \cdot \frac{9}{2}$

~~$\frac{2 \cdot 2 \cdot 3 \cdot 3}{3 \cdot 5 \cdot 2}$~~ $\left(\frac{6}{5}\right)$

Simplifying Rational Expressions

- factor the numerator and denominator.
- cancel the like terms.

Multiplying Rational Expressions

- factor every numerator and denominator.
- cancel the like terms of ANY numerator with ANY denominator.
- multiply like fractions.

Dividing Rational Expressions

- multiply by the reciprocal.
- multiply.

Examples: Simplify the following rational expressions.

Determine the undefined terms.

1.
$$\frac{3y(\cancel{y+7})}{(\cancel{y+7})(\cancel{y^2-9})}$$

 $(y+3)(y-3)$

UT: $y \neq -7, \pm 3$

$$\frac{3y}{(y+3)(y-3)}$$

2.
$$\frac{-\cancel{ab}(b-\cancel{5})}{\cancel{ab^2} \quad 5ab}$$

 $(5+b)(\cancel{5-b})$

UT: $b \neq \pm 5$

$$\frac{-ab}{5+b}$$

Examples: Simplify the following rational expressions.
Determine the undefined terms.

3. $\frac{x^2 + 5x + 6}{x^2 - 4}$

$\frac{(x+2)(x+3)}{(x+2)(x-2)}$
UT. $x \neq \pm 2$

4. $\frac{-a^4(1-2)}{a^4b - 2a^4}$
 $\frac{2a^3 - a^3b}{a^3(2-b)}$

UT. $a \neq 0$ $b \neq 2$
 $\frac{-a^4(1-2)}{a^3(2-b)} = \frac{-a^4(1-2)}{a^3(2-b)} = \frac{-a(1-2)}{2-b} = \frac{a}{2-b}$

Examples: Simplify the following rational expressions.
Determine the excluded values.

5. $\frac{x^2 + 9x + 20}{x^2 - 16} \cdot \frac{2x - 8}{x^2 - 25}$

6. $\frac{2}{x-5} \cdot \frac{3}{4}$
 $\frac{2 \cdot 3}{4(x-5)} = \frac{3}{2(x-5)}$

7. $\frac{x+2}{x+3} \div \frac{x^2+x-12}{x^2-9}$
 $\frac{x+2}{x+3} \cdot \frac{x^2-9}{x^2+x-12}$
 $\frac{(x+2)(x+3)(x-3)}{(x+3)(x+4)(x-3)}$
 $\frac{x+2}{x+4}$
 $x=0 \Rightarrow \frac{1}{2}$
 $x=1 \Rightarrow \frac{3}{5}$
 $\frac{3}{4} \div \frac{-10}{-8} = \frac{3}{4} \cdot \frac{8}{10} = \frac{24}{40} = \frac{3}{5}$

Complex Fraction: a rational expression with a numerator and/or denominator that is also a rational expression.

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Simplify.

8.
$$\frac{\frac{x^2}{9x^2 - 4y^2}}{\frac{x^3}{2y - 3x}}$$

Handwritten work for problem 8:

$$\frac{x^2}{9x^2 - 4y^2} \cdot \frac{(-2y + 3x)}{x^3}$$

$$\frac{x^2}{(3x+2y)(3x-2y)} \cdot \frac{-1 \cdot x^2}{x^3}$$

$$\frac{-1 \cdot x \cdot x}{x \cdot x \cdot x (3x+2y)}$$

$$\frac{-1}{x(3x+2y)}$$

