8-4A Graphs of Rational Functions

Rational Function: an equation of the form $f(x) = \frac{p(x)}{q(x)}$,
where $p(x)$ and $q(x)$ are polynomials
and $q(x) \neq 0$.

Examples

1. $f(x) = \dfrac{x^2 + 5x + 6}{x^2 - 4}$

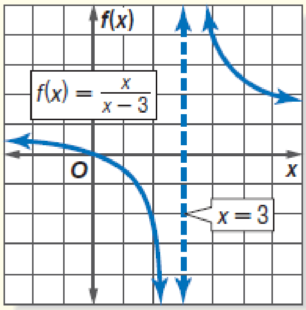
2. $g(x) = \dfrac{x^2 + 6x + 9}{x + 3}$

Rational Function Graphs: there may be breaks in continuity *(graph cannot be traced without lifting the pencil).*

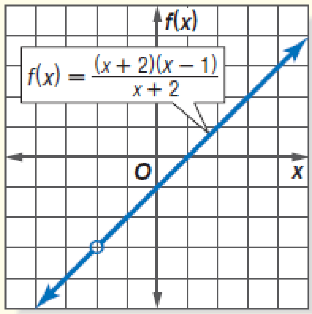
2 Types of Breaks in Continuity

- **vertical asymptotes**
- **point discontinuity (holes)**

Key Concept — *Vertical Asymptotes*

Property	Words	Example	Model
Vertical Asymptote	If the rational expression of a function is written in simplest form and the function is undefined for $x = a$, then $x = a$ is a vertical asymptote.	For $f(x) = \frac{x}{x-3}$, $x = 3$ is a vertical asymptote.	

Key Concept — *Point Discontinuity*

Property	Words	Example	Model
Point Discontinuity	If the original function is undefined for $x = a$ but the rational expression of the function in simplest form is defined for $x = a$, then there is a hole in the graph at $x = a$.	$f(x) = \frac{(x+2)(x-1)}{x+2}$ can be simplified to $f(x) = x - 1$. So, $x = -2$ represents a hole in the graph.	

Examples: *Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of the following rational functions.*

1. $f(x) = \dfrac{x^2 - 4}{x^2 + 5x + 6}$

2. $g(x) = \dfrac{x}{x^2 - 3x}$

3. $h(x) = \dfrac{5}{x^2 - 16}$