

## 8-4A Graphs of Rational Functions

**Rational Function:** an equation of the form  $f(x) = \frac{p(x)}{q(x)}$ , where  $p(x)$  and  $q(x)$  are polynomials

and  $q(x) \neq 0$ .

$$f(x) = \frac{x+3}{x-2} \quad f(x) = \frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)}$$

$$g(x) = \frac{\cancel{(x+3)}(x+3)}{\cancel{(x+3)}}$$

**Examples**

1.  $f(x) = \frac{x^2 + 5x + 6}{x^2 - 4}$

$$\frac{(x+2)(x+3)}{(x+2)(x-2)}$$

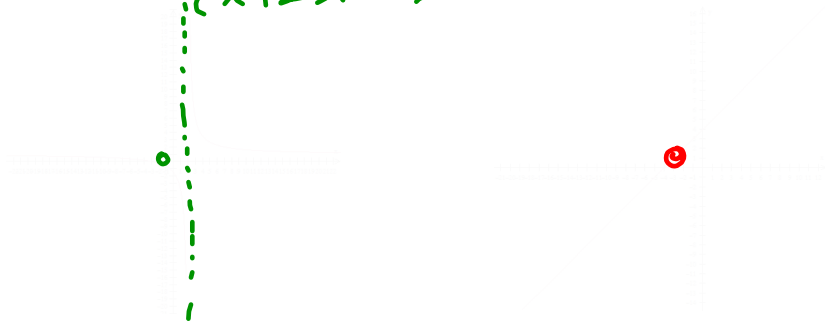
$$x \neq 2$$

$$x \neq -2$$

2.  $g(x) = \frac{x^2 + 6x + 9}{x + 3}$

$$x \neq -3$$

$$g(x) = x + 3$$



**Rational Function Graphs:** there may be breaks in continuity (graph cannot be traced without lifting the pencil).

### 2 Types of Breaks in Continuity (excluded values)

- vertical asymptotes if excluded value remains in simplified function
- point discontinuity (holes) no longer excluded in simplified function

$$\frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)}$$

↓  
hole

↓  
v.A.

**Examples:** Determine the equations of any vertical asymptotes and the values of  $x$  for any holes in the graph of the following rational functions.

$$1. f(x) = \frac{x^2 - 4}{x^2 + 5x + 6}$$

$$f(x) = \frac{\cancel{(x+2)}(x-2)}{\cancel{(x+2)}(x+3)}$$

$$\text{V.A. } x = -3$$

$$\text{hole: } x @ -2$$

$(-2, -)$

$$2. g(x) = \frac{x}{x^2 - 3x}$$

$$g(x) = \frac{\cancel{x}}{\cancel{x}(x-3)}$$

$$\text{V.A. } x = 3$$

$$\text{hole @ } 0$$

$$3. h(x) = \frac{5}{x^2 - 16}$$

$$h(x) = \frac{5}{(x+4)(x-4)}$$

$$\text{V.A. } x = -4$$

$$x = 4$$

$$\text{hole: none}$$